Normal Form Analysis of the Tevatron*

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Abstract

A systematic analysis effort is under way to calculate and model the effects of present and future upgrades to the Tevatron. As a first step, using normal form theory, amplitude-dependent tuneshifts for current Tevatron collider operating conditions were calculated. The normal form algorithm was implemented within the framework of the object-oriented accelerator physics class libraries BEAMLINE[1] and MXYZPTLK[2] under development at Fermilab.

1 INTRODUCTION

The goal of the Fermilab collider program is to increase collider luminosities each run until peak luminosities of $> 8.0 \times 10^{31}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ and integrated luminosities of $100\,\mathrm{pb}^{-1}$ per run are achieved. To achieve these goals requires a number of accelerator upgrades. One of the major upgrades to the Tevatron was the addition of electrostatic separators which were added before last collider run. The use of separators has been very successful in lowering the beam-beam tune shift. However the closed orbit for each beam is no longer through the center of the magnets. Since the protons and antiprotons are traveling on different orbits, they experience different nonlinear fields and therefore have different tunes coupling and chromaticities. Groups of sextupoles can be used to correct the differential effects. Although extensive investigation of these effects was done during the design stage, over time the operation of the Tevatron changes and over time the beam intensity increases requiring more stringent control and understanding of the accelerator.

Nonlinear methods have been used in the design of the SSC and currently the LHC [3] but have not been applied to the Tevatron. An object-oriented accelerator physics toolkit [4] in C++ is under development to address this and other accelerator design projects. Most of the linear part of the toolkit has been developed. A normal form algorithm[5] has been implemented as the first nonlinear part of the toolkit. This algorithm was used to calculate the amplitude-dependent tune shift for the current Tevatron lattices.

2 CALCULATION METHOD

Any realistic model of the Tevatron lattice is a nonlinear one because of the sextupoles, multipole errors in the magnets and the use of electrostatic separators. Normal form theory [6] is a useful tool for calculating nonlinear parameters such as amplitude-dependent tune shifts, fixed point locations, and chromaticities. Normal form algorithms have been implemented by various authors [7, 8].

The goal of the normal form analysis is to take a complicated one-turn transfer map of the accelerator and transform if by some nonlinear transformation into "simple" (normal form) mad and analyze the simpler map. The ultimate goal is of course a linear map. The new coordinates are the eigencoordinates of the linear part of the original map and are complex. The C++ language made the implementation of complex automatic differentiation very easy because new types can be created which behave in every respect like fully functional variables of the language. The whole normal form algorithm is about sixty lines of code.

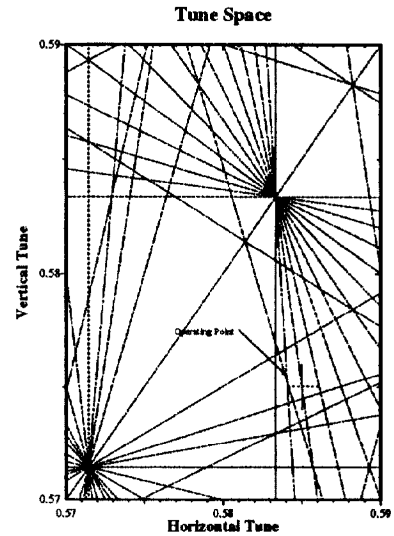

Figure 1: Resonance Diagram Showing the Tevatron Operating Point.

The desired operating point for the Tevatron is 20.585 for the horizontal plane and 20.575 for the vertical plane. It has been observed in collider operations that a 7th order resonance (20.5714) is strong enough to cause problems. A 12th order resonance (20.5833) also has some effect on the beam. Figure 1 shows the tune operating point with sum and difference resonances drawn to twelveth order. It is therefore useful to calculate which amplitudes will be tune-shifted onto these resonances.

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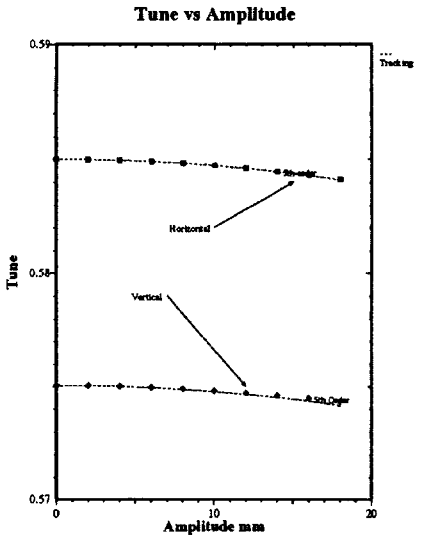

Figure 2: Comparison of Tracking with the Normal Form Calculation.

During the collider filling process, the lattice changes configuration several times. At the injection energy of 150 GeV and a $\beta^* = 1.7$m, protons are injected onto a central orbit with the separators at zero voltage. The separators are turned on and the antiprotons are injected onto the helical orbit. after both beams are accelerated to 900 GeV the β^* is changed to 0.35 m, and the beams are brought into collision at only the interaction regions. Because of head-tail instability problems the chromaticity for both planes is maintained at a large positive values of greater than ten units.

The tune shift vs. amplitude was calculated for the various lattice configurations. The results were checked with tracking. Figure 2 shows a comparison of tracking and a 5th order map. In a 7th order map there were indications of resonance terms. A resonant normal form algorithm is under development. For this paper only the non-resonant normal form was used. The 5th order map was used for all calculations.

3 RESULTS

Using the emittances from a typical collider run the amplitude-dependent tune shifts were calculated for lattices in the collider fill sequence. The chromaticity was fixed at ten units for each plane by adjusting the trim sexupoles. The beam σ is typically 1.2mm at 150 GeV and .6 mm at 900 GeV. A tune "footprint" was calculated by holding the horizontal amplitude fixed and varying the vertical amplitude and vice versa. The amplitudes were varied from 0.0 to 10 mm in steps of 1mm. Figure 3 shows the tune shift footprint super-imposed over the resonance diagram for the injection $\beta^* = 1.7$m. As can be seen from the figure, the resonance at .5833 is crossed at amplitudes of $\approx$ 8mm; much larger that the normal beam size. Figure 4 shows the tune shift footprint super-imposed over the resonance diagram for $\beta^* = .35$m. lattice The tune shift is much larger at this value of β^* as expected. Particles at $\approx 5\sigma$ have a tune close to the 7th order resonance of .5714.

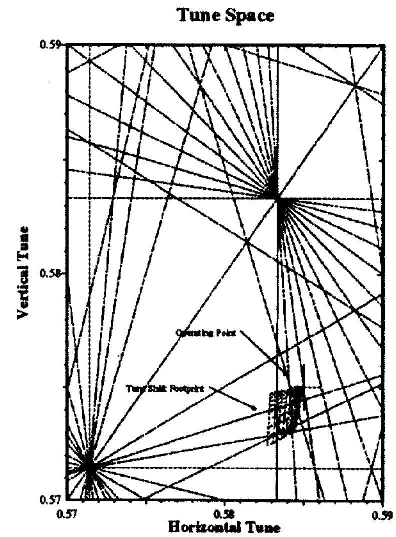

Figure 3: Tune Shift vs. Amplitude for $\beta^* = 1.7$m.

For helical orbits, the only difference in the calculation was to subtract the non-zero fixed point from the map before calculating the normal form. Figure 5 shows the tune shift footprint super-imposed over the resonance diagram for the $\beta^* = .35$m and a helical orbit with 5σ separation. The local separator bumps are added so that the proton and antiproton orbits coincide at the two experimental regions. The helix changes the shape of the footprint slightly and the tune shift is slightly less.

The lattice magnet strengths were taken from operational data. Before installation, the Tevatron magnet multipole errors were measured. These errors will be added to the lattice in the future. A large source of tune shift is of course the beam-beam tune shift. Development is underway to incorporate beam-beam effects into our beamline class library.

4 CONCLUSION

As part of a systematic analysis effort a normal form algorithm has been added to the MXYZPTLK class library. Amplitude dependent tune shifts have been calculated for the current Tevatron lattices. Under typical operatating conditions and beam sizes the tune shift due to the sextupoles is only large enough to cross resonances which are known to be a problem for particles greater than 5σ. Further work is under way to implement a resonant normal form algorithm and include beam-beam effects.

5 ACKNOWLEDGEMENTS

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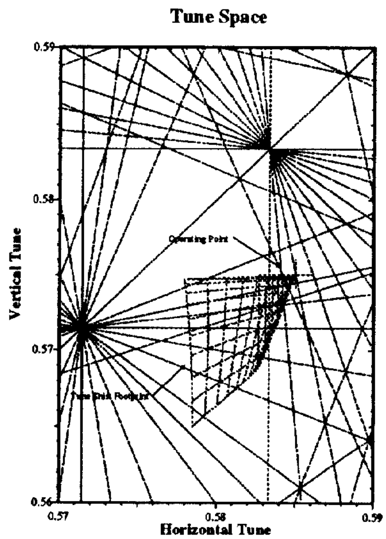

Figure 4: Tune Shift vs. Amplitude for $\beta^* = .35$m.

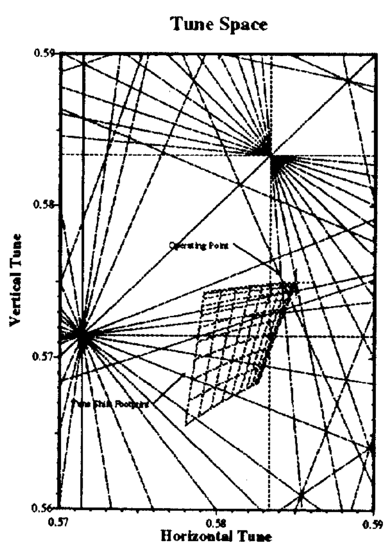

Figure 5: Tune shift vs. Amplitude for $\beta^* = .35$m helical orbit.

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